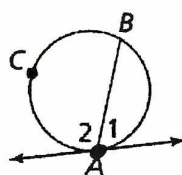


Angle Relationships in Circles

Learning Objective | WBAT find measures of $\angle$ s and intercepted arcs formed by chords, tangents, and secants.

Tangent and Intersected Chord Theorem

If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is half the measure of its intercepted arc.



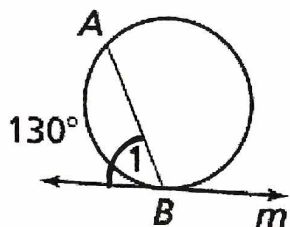
$$m\angle 1 = \frac{1}{2} m\widehat{AB}$$

$$m\angle 2 = \frac{1}{2} m\widehat{ACB}$$

Example:

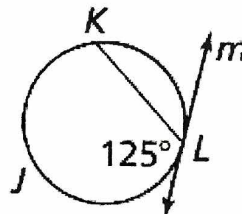
1. Line m is tangent to the circle. Find the measure of each of the following...

a. $m\angle 1$



$$m\angle 1 = \frac{1}{2} (130) = 65^\circ$$

b. $m\widehat{KJL}$

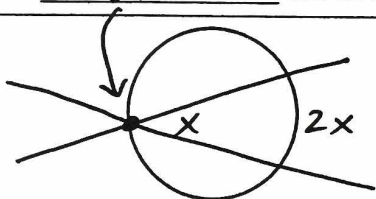


$$250^\circ$$

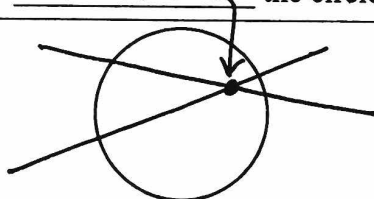
Intersecting Lines and Circles

If two nonparallel lines intersect a circle, there are three places where the lines can intersect

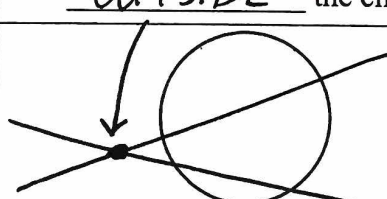
ON the circle



INSIDE the circle

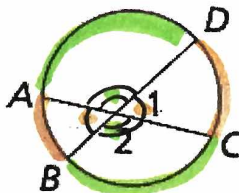


OUTSIDE the circle



Angles Inside the Circle Theorem

If two chords intersect *inside* a circle, then the measure of each angle is half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

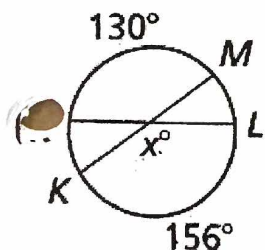


$$m\angle 1 = \frac{1}{2} (m\widehat{DC} + m\widehat{AB})$$

$$m\angle 2 = \frac{1}{2} (m\widehat{BC} + m\widehat{AD})$$

Examples:

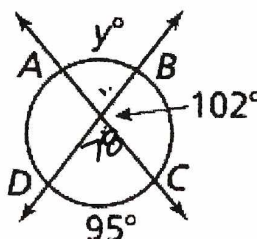
2. Find the value of x .



$$x = \frac{1}{2} (156 + 130)$$

$$x = 143$$

3. Find the value of y .



$$78 = \frac{1}{2} (95 + y)$$

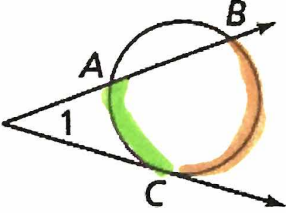
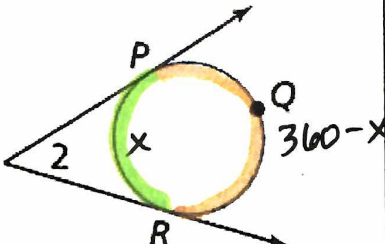
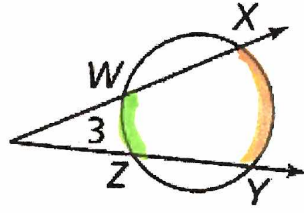
$$156 = 95 + y$$

$$y = 61$$

Angles Outside the Circle Theorem

$$m\angle = \frac{1}{2}(\text{outer arc} - \text{inner arc})$$

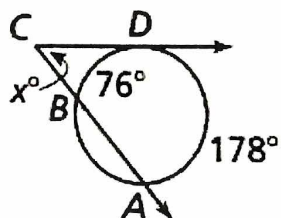
If a tangent and a secant, two tangents, or two secants intersect *outside* a circle, then the measure of the angle formed is half the difference of the measures of the intercepted arcs.

Tangent and Secant	Two Tangents	Two Secants
		
$m\angle 1 = \frac{1}{2}(m\widehat{BC} - m\widehat{AC})$	$m\angle 2 = \frac{1}{2}(m\widehat{PQR} - m\widehat{PR})$	$m\angle 3 = \frac{1}{2}(m\widehat{XY} - m\widehat{WZ})$

Circumscribed Angle: an angle whose sides are tangent to a circle

Examples:

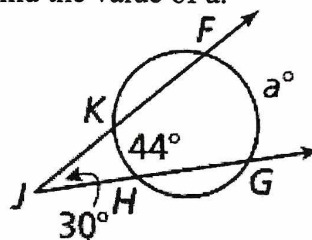
4. Find the value of x .



$$x = \frac{1}{2}(178 - 76)$$

$$\boxed{x = 51}$$

5. Find the value of a .

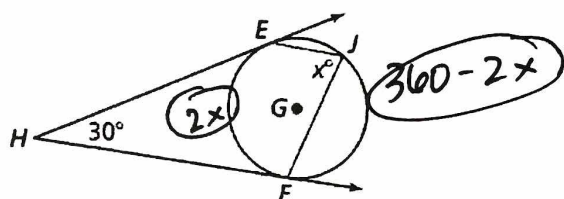


$$30 = \frac{1}{2}(a - 44)$$

$$60 = a - 44$$

$$\boxed{a = 104}$$

6. Find the value of x .

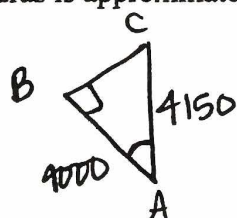


$$30 = \frac{1}{2}(360 - 2x - 2x)$$

$$60 = 360 - 4x$$

$$\boxed{75 = x}$$

7. The northern lights are bright flashes of colored light between 50 and 200 miles above the earth. A flash occurs 150 miles above Earth at point C. What is the measure of $\widehat{BD}$, the portion of the Earth from which the flash is visible? (Earth's radius is approximately 4000 miles)



$$m\angle A = \cos^{-1}\left(\frac{4000}{4150}\right)$$

$$m\angle A \approx 15.45^\circ$$

$$\Rightarrow m\angle BAD \approx 30.90^\circ \Rightarrow \boxed{m\widehat{BD} \approx 30.90^\circ}$$

